Your Name

Student ID #

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- Check that you have a complete exam. There are 5 questions for a total of 47 points.
- You are allowed to have one 8.5" $\times$ 11" handwritten note sheet, both sides, and a TI-30X IIS basic scientific calculator
- Cheating will result in a zero and be reported to the Dean's Academic Conduct Committee.
- **Show all your work**. Unless explicitly stated otherwise in a particular question, if there is no work supporting your answer, you will not receive credit for the problem. If you need more space to answer a question, continue on the back of the page, and *indicate that you have done so.*

Question	Points	Score
1	9	
2	9	
3	10	
4	12	
5	7	
Total:	47	

1. (a) (1 point) Expand $(A+B)(A-B)$.

   (b) (1 point) What is nullity(0_{mn}), where 0_{mn} is the $m \times n$ matrix with all 0 entries?

   (c) (4 points) Define 2 of the following 3 terms: linear transformation, subspace, onto.

   (d) (3 points) Describe the linear transformation with matrix

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 3 \end{pmatrix}$$

   geometrically. (Pictures encouraged.)

2. (a) (4 points) Compute the inverse of the following matrix, or show it does not exist:

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

(b) (5 points) Express $\operatorname{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\} \subset \mathbb{R}^4$ as the solution set of a linear system.

3. (a) (3 points) Give an example of a matrix A where $A^6 = I_2$ yet no smaller (positive) power of A is I_2.

   (b) (3 points) Give an example of a linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ where $\ker T = \operatorname{range} T$.

   (c) (4 points) Give an example of linear transformations $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ and $U\colon \mathbb{R}^3 \to \mathbb{R}^2$ such that (i) T is one-to-one, (ii) U is onto, and (iii) the composite $U \circ T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is neither one-to-one nor onto.

4. In the following, assume you have access to a computer to perform computations like Gauss–Jordan elimination or standard matrix operations.

   (a) (6 points) Give a **step-by-step recipe** for determining if two sets of vectors span the same subspace of $\mathbb{R}^n$. Your answer should be detailed enough that your classmates could follow the recipe without asking you for more details.

   (b) (6 points) You are given a basis $\vec{u}_1, \vec{u}_2, \vec{u}_3$ for $\mathbb{R}^3$ along with some additional vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3 \in \mathbb{R}^4$. Give a **step-by-step recipe** for computing the matrix of the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ for which $T(\vec{u}_1) = \vec{b}_1$, $T(\vec{u}_2) = \vec{b}_2$, and $T(\vec{u}_3) = \vec{b}_3$.

5. A linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ has the following matrix A with reduced echelon form B:

$$A = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -1 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = B.$$

(a) (4 points) Give bases for **both** row(A) and col(A).

(b) (3 points) Is T one-to-one, onto, and/or invertible?