Your Name

Student ID #

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- Check that you have a complete exam. There are 4 questions for a total of 42 points.
- You are allowed to have one 8.5" × 11" handwritten note sheet, both sides, and a TI-30X IIS basic scientific calculator
- Cheating will result in a zero and be reported to the Dean's Academic Conduct Committee.
- **Show all your work**. Unless explicitly stated otherwise in a particular question, if there is no work supporting your answer, you will not receive credit for the problem. If you need more space to answer a question, continue on the back of the page, and *indicate that you have done so.*

Question	Points	Score
1	10	
2	12	
3	8	
4	12	
Total:	42	

1. (a) (1 point) Write $\begin{pmatrix} \pi \\ 3 \\ 1 \\ 4 \end{pmatrix}$ as a linear combination of standard basis vectors.

(b) (3 points) (Check all that apply.) Which of the following statements are *always true*? Assume dimensions are compatible.

◯ $A(\vec{x} + \vec{y}) = A\vec{x} + A\vec{y}$ ◯ $A(3\vec{x}) = 3(A\vec{x})$ ◯ if $A\vec{x} = \vec{b}$, then $\vec{x} = \frac{\vec{b}}{A}$
◯ $(a+b)(\vec{u}+\vec{v}) = a\vec{u} + b\vec{v}$ ◯ $[\vec{a}_1 \ \cdots \ \vec{a}_m]\vec{e}_1 = \vec{a}_1$

(c) (4 points) Define 2 of the following 3 terms: trivial linear combination; coefficient matrix; span.

(d) (2 points) Recall that the set of unit vectors in $\mathbb{R}^3$ is

$$\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}.$$

Can the set of unit vectors in $\mathbb{R}^3$ be written as the span of some vectors? If so, what vectors? If not, why not?

2. Consider the linear system

$$\begin{aligned} 3x_1 + 3x_2 + x_3 + 2x_4 + 2x_5 &= 0 \\ x_1 + x_2 + x_3 + 2x_4 + 2x_5 &= 0 \\ x_1 + x_2 \phantom{{}+x_3} + x_4 + 2x_5 &= 0 \end{aligned}$$

(a) (2 points) Write the linear system in the form $A\vec{x} = \vec{b}$.

(b) (7 points) Give the general solution of the linear system.

(c) (3 points) Express the solution set as the span of some vectors.

3. Provide examples meeting the following specifications.

   (a) (2 points) An inconsistent linear system whose augmented matrix is in reduced echelon form.

   (b) (3 points) Vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$ which are linearly *dependent* but where $\vec{u}_1$ is not in $\text{span}\{\vec{u}_2, \vec{u}_3\}$.

   (c) (3 points) Vectors $\vec{u}_1, \vec{u}_2, \vec{v}_1, \vec{v}_2 \in \mathbb{R}^3$ where the only vector in both $\text{span}\{\vec{u}_1, \vec{u}_2\}$ and $\text{span}\{\vec{v}_1, \vec{v}_2\}$ is $\vec{0}$.

4. Describe *step-by-step* recipes for solving the following problems if you had access to a computer which could do basic operations like row reduction, vector arithmetic, etc. "Pseudocode" descriptions are encouraged. Do not simply provide a single example; describe the general procedure. **Briefly justify** your procedure.

   (a) (4 points) Given a linear system, describe how to determine if it has 0, 1, or infinitely many solutions.

   (b) (4 points) Given $\vec{u}_1, \ldots, \vec{u}_m \in \mathbb{R}^n$, describe how to determine if they span $\mathbb{R}^n$.

   (c) (4 points) Given two linear systems with the same variables, describe how to determine whether or not they have the same solution set. (Hint: be careful about the number of equations and inconsistent systems.)