

Math 308 M – Spring 2015
Proof Homework 1
Due Wednesday, April 15th, 2015

Name: _____

- **Answer two** of the following three questions. If you answer them all, your *best* answer will be ignored.
- Give rigorous proofs. Any skipped steps must be small enough that you could explain them to me in a few seconds. Your goal is to convince me you fully understand your argument and have not missed anything.
- You may use any theorem, proposition, etc. from lecture or the book, though when you do say at least “from the book” or “from lecture.”
- For examples to model your proofs on, see the textbook, the proof examples document on the course web site, or any of the alternatives to the textbook linked from the course web site.
- You are welcome to talk to others (even outside the class) or work in groups on this assignment, though write your final answers alone. Keep in mind that this exercise is entirely for your benefit in becoming more comfortable with proofs.

1. Write down all the reduced row echelon forms for 2×3 matrices. How many distinct sets of possible pivot positions are there? (See Corollary 2 of the proof examples document for an example proof by cases.)

2. Show that the system of equations

$$a_{11}x_1 + a_{12}x_2 = 0$$

$$a_{21}x_1 + a_{22}x_2 = 0$$

has more than one solution if and only if $a_{11}a_{22} = a_{12}a_{21}$. (Hint: row reduction. You may also solve the easier version where each a_{ij} is non-zero.)

3. Suppose the solutions of a homogeneous linear system of two equations in three variables are all the multiples of $(-3, 1, 0)$, i.e. are precisely of the form $x_1 = -3s, x_2 = s, x_3 = 0$ for arbitrary s . Show that the corresponding augmented matrix's reduced row echelon form is

$$\begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

(You may either follow the challenge problem outline from class, follow the proof of Theorem 6 from the proof examples document, or create your own proof. If you create your own proof, you may assume there are two pivots, one in the first column and the other in the third column. Extra credit will be given to anyone who produces an original argument which includes the computation of pivot positions and number.)