MATH 308 H
Exam I
Autumn 2013

Name ______________________________

Student ID #__________________

Section ____________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

1	20	
2	10	
3	10	
4	10	
Total	50	

- Your exam should consist of this cover sheet, followed by 4 problems. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 4 problems. Try not to spend more than about 12 minutes on each page.
- Unless otherwise indicated, show all your work and justify your answers.
- Unless otherwise indicated, your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- Do not use scratch paper. Put all your work on the exam. If you run out of room, use the back of the page and indicate to the reader you have done so.
- You may use a scientific calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (20 points) Indicate whether the statement is true (T) or false (F). Circle your response.

   (a) A homogeneous linear system may have exactly one non-trivial solution.

   ANSWER: (circle one) T F

   (b) A linear system with more variables than equations must have infinitely many solutions.

   ANSWER: (circle one) T F

   (c) A set of two vectors is linearly dependent if and only if one is a scalar multiple of the other.

   ANSWER: (circle one) T F

   (d) The following matrix is in reduced echelon form.

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

   ANSWER: (circle one) T F

   (e) If A is a matrix with more columns than rows, then the columns of A form a linearly independent set.

   ANSWER: (circle one) T F

   (f) Every vector in $span\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is in $span\{\mathbf{u}_1, \mathbf{u}_2\}$.

   ANSWER: (circle one) T F

   (g) If $\mathbf{v}$ is a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$, then $\mathbf{v}$ is also a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

   ANSWER: (circle one) T F

   (h) If $\{\mathbf{u}_1, \mathbf{u}_2\}$ is linearly independent and $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is linearly dependent, then $\mathbf{u}_3$ is in $span\{\mathbf{u}_1, \mathbf{u}_2\}$.

   ANSWER: (circle one) T F

   (i) If $m > n$, any set of m vectors in $\mathbb{R}^n$ spans $\mathbb{R}^n$.

   ANSWER: (circle one) T F

   (j) Any linearly independent set of n vectors in $\mathbb{R}^n$ spans $\mathbb{R}^n$.

   ANSWER: (circle one) T F

2. (10 points) Use any valid method to solve the system. Place your final answer in the blank below.

$$\begin{aligned} 2x_1 + x_2 + x_3 + x_4 &= 2 \\ -x_1 - 2x_3 + x_4 &= 5 \\ x_2 + 4x_3 + 17x_4 &= 5 \end{aligned}$$

ANSWER: $(x_1, x_2, x_3, x_4) =$ ______________________________

3. (10 points)

(a) Find all values of h so that the columns of the following matrix span $\mathbb{R}^2$.

$$\begin{bmatrix} 4 & 10 \\ 3 & h \end{bmatrix}$$

(b) Find all values of k so that the following linear system has infinitely many solutions.

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 + 4x_2 - x_3 &= k \\ 2x_1 - x_2 + 4x_3 &= k^2 \end{aligned}$$

4. (10 points) Let $\mathbf{u}_1 = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 8 \\ 5 \\ -3 \end{bmatrix}$.

   (a) Let $\mathbf{v} = \begin{bmatrix} 26 \\ 17 \\ -8 \end{bmatrix}$. Write $\mathbf{v}$ as a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$.

   (b) Give an example of a vector $\mathbf{w}$ that is not in $span\{\mathbf{u}_1, \mathbf{u}_2\}$. You must show some work to justify your answer.