MATH 308 H
Final Exam
Autumn 2013

Name ______________________________ Student ID #_______________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

1	20	
2	12	
3	12	
4	6	
5	15	
6	15	
Total	80	

- Your exam should consist of this cover sheet, followed by 6 problems on 7 pages. Check that you have a complete exam.
- Unless otherwise indicated, show all your work and justify your answers.
- Unless otherwise indicated, your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- Do not use scratch paper. Put all your work on the exam. If you run out of room, use the back of the page and indicate to the reader you have done so.
- You may use a scientific calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing and programmable calculators) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (20 points) Indicate whether the statement is true (T) or false (F). Circle your response. You are not required to show any work.

   (a) Applying row operations does not change the eigenvalues of a matrix.

   ANSWER: (circle one) T F

   (b) Any vector $\mathbf{x}$ with the property that $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ is an eigenvector of A.

   ANSWER: (circle one) T F

   (c) If $A = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$, then $\mathbf{v}$ is an eigenvector of A.

   ANSWER: (circle one) T F

   (d) If $\mathbf{u}_1$ and $\mathbf{u}_2$ are eigenvectors of a matrix A and $\{\mathbf{u}_1, \mathbf{u}_2\}$ is linearly independent, then $\mathbf{u}_1$ and $\mathbf{u}_2$ must correspond to different eigenvalues.

   ANSWER: (circle one) T F

   (e) Every basis of $\mathbb{R}^2$ is an orthogonal basis.

   ANSWER: (circle one) T F

   (f) If S is a subspace of $\mathbb{R}^3$ with dimension 1, then so is $S^\perp$.

   ANSWER: (circle one) T F

   (g) If $\mathbf{u}$ is orthogonal to $\mathbf{v}$, then $||\mathbf{u} + \mathbf{v}|| = ||\mathbf{u} - \mathbf{v}||$.

   ANSWER: (circle one) T F

   (h) The zero vector in $\mathbb{R}^n$ is orthogonal to every vector in $\mathbb{R}^n$.

   ANSWER: (circle one) T F

   (i) If $\mathbf{u}$ is in the subspace S, then $\text{proj}_S\mathbf{u} = \mathbf{u}$.

   ANSWER: (circle one) T F

   (j) If A is any 2×2 diagonalizable matrix with eigenvalues $\lambda = 1$ and $\lambda = -1$, then $A^k = A$ for every positive integer k.

   ANSWER: (circle one) T F

2. (12 points) Give an example of each of the following. You are not required to show any work.

   (a) a 4×4 matrix with determinant 12

   (b) a basis $\mathcal{B}$ of $\mathbb{R}^2$ such that
   $$\begin{bmatrix} 4 \\ 3 \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$

   (c) an orthogonal set of three vectors in $\mathbb{R}^3$, none of which is a scalar multiple of any of the standard basis vectors

   (d) the characteristic equation for a matrix that has exactly two real eigenvalues, one with multiplicity 2 and the other with multiplicity 4

3. (12 points) Find the characteristic polynomial, the eigenvalues, and a basis for each eigenspace of the matrix A. Mark your responses clearly.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

4. (6 points) Let $S = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$, where $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.

   Find $\text{proj}_S \mathbf{u}$ if $\mathbf{u} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$.

5. (15 points) Find the value of k so that the matrix A is diagonalizable and then find the matrices P and D such that $A = PDP^{-1}$.

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}.$$

6. (15 points) Consider $\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$, a basis for $\mathbb{R}^3$.

(a) Compute $\mathbf{x}_{\mathcal{B}_1}$ if $\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$.

THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.

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(b) Let $\mathcal{B}_2 = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$. (Note that $\mathcal{B}_2$ is also a basis for $\mathbb{R}^3$.)

Find the change of basis matrix from $\mathcal{B}_2$ to $\mathcal{B}_1$.