

Math 20C – Swanson – Winter 2021  
Homework 8  
Due Sunday, 3/7/21 at 11:59pm

- The graded part of the homework is on WebAssign.
- The problems below are also assigned and you are responsible for doing them, but they will not be collected or graded.

1. Let  $\vec{r}(t)$  be a differentiable path in  $\mathbb{R}^3$ . Use the differentiation rules on page 218 to evaluate the following in terms of  $\vec{r}(t)$  and its derivatives.
- (a) Evaluate...
- (i)  $\frac{d}{dt}(t^2\hat{i} \times \vec{r}(t))$
  - (ii)  $\frac{d}{dt}(t^2\hat{i} \cdot \vec{r}(t))$
  - (iii)  $\frac{d}{dt}(t^2\vec{r}(t))$
  - (iv)  $\frac{d}{dt}(5t^2\hat{i} + 3\vec{r}(t))$
  - (v)  $\frac{d}{dt}\vec{r}(t^2)$
- (b) Evaluate  $\frac{d}{dt}\|\vec{r}(t)\|$ . (Hint: recall that  $\|\vec{r}(t)\|^2 = \vec{r}(t) \cdot \vec{r}(t)$ .)
- (c) Evaluate and simplify  $\frac{d}{dt}(\vec{r}(t) \times \vec{r}'(t))$ .
- (d) Prove that if the speed is constant, i.e.  $\|\vec{r}'(t)\| = k$  for some constant  $k$ , then the velocity and the acceleration vectors are orthogonal. (Hint: what is the value of  $\vec{r}'(t) \cdot \vec{r}'(t)$  and how is it related to the speed? Differentiate this expression.)

2. A particle travels on the path  $\vec{c}(t) = e^t\hat{i} + e^{-t}\hat{j} - \sqrt{2}t\hat{k}$ . Find the arc length between  $e^{-3}\hat{i} + e^3\hat{j} + 3\sqrt{2}\hat{k}$  and  $e^2\hat{i} + e^{-2}\hat{j} - 2\sqrt{2}\hat{k}$ . (Hint: in computing the speed, you will get the square root of a perfect square of a function. Simplify to get a function that's easy to integrate.)