Math 20C – Swanson – Winter 2021
Homework 5
Due Tuesday, 2/16/21 at 11:59pm

- The graded part of the homework is on WebAssign.
- The problems below are also assigned and you are responsible for doing them, but they will not be collected or graded.

1. Sketch these parametric curves by hand, using methods discussed in class for Chapter 2.4:

   (a) $\vec{c}(t) = (t\cos(t), t\sin(t))$, where $0 \le t \le 4\pi$.

   (b) $x = -3t\sin(t)$, $y = 4t\cos(t)$, where $0 \le t \le 4\pi$.

   (c) $\vec{c}(t) = -\sin^2(t)\hat{\imath} + \cos^2(t)\hat{\jmath}$.

   (d) $\vec{c}(t) = 2t\hat{\imath} + 3\cos(t)\hat{\jmath} + 4\sin(t)\hat{k}$.

   (e) $x = \cos(t), y = \sin(t), z = \sin(3t)$.

   (f) $x = 5e^{-t}, y = e^{t}$, where $0 \le t \le \ln(10)$.

   After sketching them by hand, check them with a computer.

2. Recall the first part of the Fundamental Theorem of Calculus: if $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$. For example, if $F(x) = \int_a^x \sin(\sqrt{t+8})\,dt$, then $F'(x) = \sin(\sqrt{x+8})$. The integral $\int_0^x e^{-t^2}\,dt$ arises in probability and statistics. It can't be evaluated in terms of ordinary functions. Nonetheless, we may compute:

   (a) $\frac{d}{dx}\int_0^x e^{-t^2}\,dt$.

   (b) $\frac{d}{dx}\int_0^{x^3} e^{-t^2}\,dt$ by completing the following outline. Let $G(u) = \int_0^u e^{-t^2}\,dt$. Note that $\int_0^{x^3} e^{-t^2}\,dt = G(x^3)$. Use the chain rule to compute $\frac{d}{dx}G(x^3)$.

   (c) Compute $\frac{\partial}{\partial x}\int_{10y}^{x^3} e^{-t^2}\,dt$ and $\frac{\partial}{\partial y}\int_{10y}^{x^3} e^{-t^2}\,dt$. Note that $\int_{10y}^{x^3} e^{-t^2}\,dt = G(x^3) - G(10y)$, using the same $G(u)$ defined in the previous part.