Math 20C – Swanson – Fall 2020
Homework 8
Due Friday, 12/4/20 at 11:59pm

- The graded part of the homework is on WebAssign.
- The problems below are also assigned and you are responsible for doing them, but they will not be collected or graded.

1. Let $\vec{r}(t)$ be a differentiable path in $\mathbb{R}^3$. Use the differentiation rules on page 218 to evaluate the following in terms of $\vec{r}(t)$ and its derivatives.

   (a) Evaluate...

   (i) $\frac{d}{dt}(t^2\hat{\imath} \times \vec{r}(t))$

   (ii) $\frac{d}{dt}(t^2\hat{\imath} \cdot \vec{r}(t))$

   (iii) $\frac{d}{dt}(t^2\vec{r}(t))$

   (iv) $\frac{d}{dt}(5t^2\hat{\imath} + 3\vec{r}(t))$

   (v) $\frac{d}{dt}\vec{r}(t^2)$

   (b) Evaluate $\frac{d}{dt}||\vec{r}(t)||$. (Hint: recall that $||\vec{r}(t)||^2 = \vec{r}(t) \cdot \vec{r}(t)$.)

   (c) Evaluate and simplify $\frac{d}{dt}(\vec{r}(t) \times \vec{r}\,'(t))$.

   (d) Prove that if the speed is constant, i.e. $||\vec{r}\,'(t)|| = k$ for some constant k, then the velocity and the acceleration vectors are orthogonal. (Hint: what is the value of $\vec{r}\,'(t) \cdot \vec{r}\,'(t)$ and how is it related to the speed? Differentiate this expression.)

2. A particle travels on the path $\vec{c}(t) = e^t\hat{\imath} + e^{-t}\hat{\jmath} - \sqrt{2}t\hat{k}$. Find the arc length between $e^{-3}\hat{\imath} + e^3\hat{\jmath} + 3\sqrt{2}\hat{k}$ and $e^2\hat{\imath} + e^{-2}\hat{\jmath} - 2\sqrt{2}\hat{k}$. (Hint: in computing the speed, you will get the square root of a perfect square of a function. Simplify to get a function that's easy to integrate.)