Math 20C – Swanson – Fall 2019
Homework 7
Due Tuesday, 11/19/19 at 11:59pm

- The graded part of the homework is on WebAssign.
- The problems below are also assigned and you are responsible for doing them, but they will not be collected or graded.

1. We will study an example where mixed partial derivatives are *not* equal at a particular point. This is based on problem 3.1.32 in the book. Consider

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Compute $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial^2 f}{\partial y\,\partial x}$. For $(x,y) \neq (0,0)$, the two mixed partials should be equal.

   (b) Evaluate the limit as $(x,y) \to (0,0)$ for each of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial^2 f}{\partial y\,\partial x}$.

   (c) It turns out that $\frac{\partial^2 f}{\partial x\,\partial y}(0,0)$ and $\frac{\partial^2 f}{\partial y\,\partial x}(0,0)$ are both defined, but are not equal! Compute them both by the following procedure, which in general computes

$$\frac{\partial^2 f}{\partial x\,\partial y} = \left.\left(\frac{\partial}{\partial x}\left(\left.\frac{\partial f}{\partial y}\right|_{y=b}\right)\right)\right|_{x=a}$$

      (i) First compute the function $\frac{\partial f}{\partial y}$.

      (ii) In computing $\frac{\partial}{\partial x}$ of a function, we hold y constant and then take the x-derivative. Hold y constant by setting $y = b$ and simplifying, which evaluates $\frac{\partial f}{\partial y}(x,b)$.

      (iii) Compute the derivative $\frac{\partial}{\partial x}$ of $\frac{\partial f}{\partial y}(x,b)$ and evaluate at $x = a$.

   (To compute the other mixed partial, swap the roles of x and y in the above procedure.)