Math 126 Spring 2010

Work with projections, dot products, and cross-products.

1. Decide for each expression below whether it is a vector (**V**), a scalar (**S**), or nonsense (**N**). Note that $\mathbf{a}, \mathbf{b}, \mathbf{u}$, and $\mathbf{v}$ are vectors, while c and d are scalars.

		Circle one:		
(a)	$\mathbf{a} \cdot (\mathbf{u} - c\mathbf{v})$	**V**	**S**	**N**
(b)	$\mathbf{a} \cdot (\mathbf{b} + c)$	**V**	**S**	**N**
(c)	$(c + d) \cdot \mathbf{a}$	**V**	**S**	**N**
(d)	$\mathbf{uv}$	**V**	**S**	**N**
(e)	$\frac{\mathbf{a}}{c}$	**V**	**S**	**N**
(f)	$\frac{c}{\mathbf{a}}$	**V**	**S**	**N**
(g)	$(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{u}$	**V**	**S**	**N**
(h)	$\mathbf{a} \times (\mathbf{b} \times \mathbf{u})$	**V**	**S**	**N**
(i)	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{u})$	**V**	**S**	**N**
(j)	$(c\mathbf{a}) \times \mathbf{b}$	**V**	**S**	**N**
(k)	$c(\mathbf{a} \cdot \mathbf{b})(\mathbf{u} \times \mathbf{v})$	**V**	**S**	**N**

2. Determine whether each of the following is true or false. If it is true, prove it. If it is false, give a counterexample. Note that $\mathbf{a}$ and $\mathbf{b}$ are vectors and c is a scalar.

   (a) Suppose $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$. Then it must be true that at least one of $\mathbf{a}$ or $\mathbf{b}$ must be the zero vector.

   (b) Suppose $c\mathbf{a} = \mathbf{0}$. Then it must be true that either $c = 0$ or $\mathbf{a} = \mathbf{0}$ (or both).

3. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors.

   (a) Show by examples that $\text{comp}_{\mathbf{a}}\mathbf{b}$ and $\text{comp}_{\mathbf{b}}\mathbf{a}$ can be the same and can be different. What conditions on $\mathbf{a}$ and $\mathbf{b}$ will guarantee they are the same?

   (b) Your friend who skips class frequently says, "I'm confused. Isn't $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$? If that is true, how can $\text{comp}_{\mathbf{a}}\mathbf{b}$ and $\text{comp}_{\mathbf{b}}\mathbf{a}$ be different?" What is your answer?

   (c) Show by examples that $\text{proj}_{\mathbf{a}}\mathbf{b}$ and $\text{proj}_{\mathbf{b}}\mathbf{a}$ can be the same and can be different. What conditions on $\mathbf{a}$ and $\mathbf{b}$ will guarantee they are the same?