Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and put away all electronic devices except your non-graphing calculator.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).
- Graphing calculators are not allowed. Do not share notes.
- In order to receive full credit, you must show all of your work on the exam paper (**even** if you could do the work in your head!). Remember to read each problem carefully and answer the questions being asked.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the **back of the previous page** and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	10	
3	10	
4	10	
5	20	

Problem	Total Points	Score
6	10	
7	8	
8	10	
9	10	
Total	100	

1. Consider the function $f(x) = \sin(x-4) + \cos(x-4) + 4\sqrt{x}$.

(a) [**6 points**] Find the second Taylor polynomial T_2 of $f(x)$ based at $b = 4$.

(b) [**2 points**] Use the second Taylor polynomial T_2 to approximate $f(4.1)$.

(c) [**4 points**] Use Taylor's inequality to find an upper bound for the error in your approximation above.

2. Consider the function $f(x) = \dfrac{1-\cos(x^2)}{x^3}$ for $x \neq 0$, and $f(0) = 0$.

(a) [**5 points**] Find the Taylor series for the function $f(x)$ about $b = 0$.
Write your answer in summation notation.

(b) [**2 points**] Find the first three nonzero terms of the Taylor series.

(c) [**3 points**] Find the interval on which the series in (a) converges.

3. Given points $P(1,0,2)$, $Q(3,-1,5)$, and $R(0,1,1)$.

(a) [**5 points**] Find the equation of the plane containing the three points P, Q, and R.

(b) [**5 points**] Write the vector $\overrightarrow{PQ}$ as a sum of two vectors, one parallel to $\overrightarrow{PR}$, and the other perpendicular to $\overrightarrow{PR}$.

4. [**10 points**] Decide if the following statements are TRUE or FALSE. You need not explain your answer.

(a) _____ The planes $2x - 2y + z = 4$ and $x - y + z = 2$ are parallel.

(b) _____ The vectors $\langle 1, -2, 5\rangle$ and $\langle 2, 1, 0\rangle$ are perpendicular.

(c) _____ If $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c}$, then $\overrightarrow{b} = \overrightarrow{c}$.

(d) _____ The lines

$$x = 2 - 3t, \quad ,y = 5 + t, \quad z = 4t$$

and

$$x = -3t, \quad y = 1 + t, \quad z = 2 + 4t$$

are parallel.

(e) _____ The line

$$x = 1 - t, \quad y = t, \quad z = 4 + 7t$$

intersects the plane $x - y - 7z = 3$ at some point.

(f) _____ The line

$$x = 2 + 2t \quad y = 5 - 8t, \quad z = -4 + 3t$$

is parallel to the plane $-x - y - 2z = 3$.

(g) _____ For any three vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$, we have $|\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c})| = |(\overrightarrow{a} \times \overrightarrow{b}) \cdot \overrightarrow{c}|$.

(h) _____ For any three vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$, we have $\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c}) = (\overrightarrow{a} \cdot \overrightarrow{b}) \times (\overrightarrow{a} \cdot \overrightarrow{c})$.

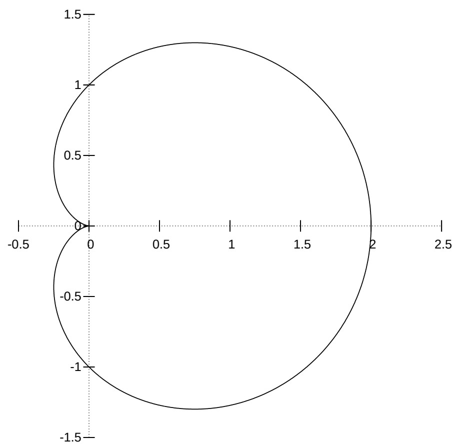
(i) _____ Two distinct lines parallel to a third line in 3D-space are parallel to each other.

(j) _____ Two distinct lines perpendicular to a third line in 3D-space are parallel to each other.

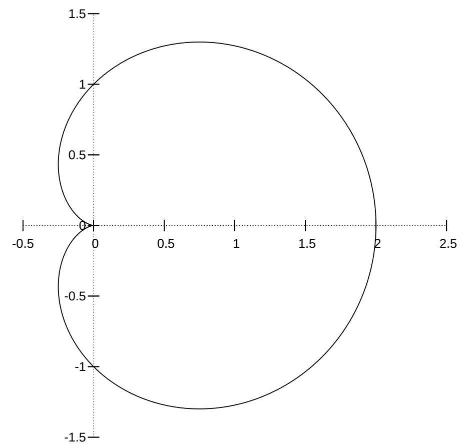
5. A curve in the xy-plane, called cardioid, is determined by the polar equation

$$r = 1 + \cos\theta.$$

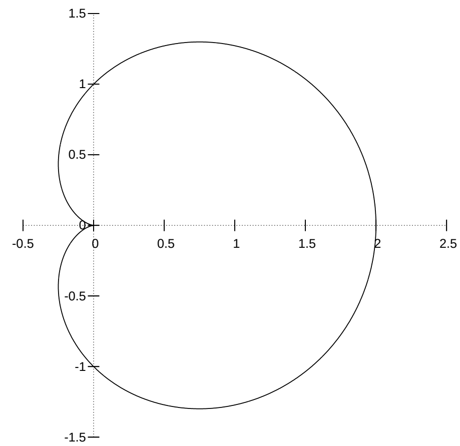
(a) [**5 points**] Find all values of θ at which the tangent line is vertical. You should find them using calculation, not the picture.



(b) [**5 points**] Find the area of the region bounded by the x-axis and the cardiod $r = 1 + \cos\theta$ from $\theta = 0$ to $\theta = \pi$.



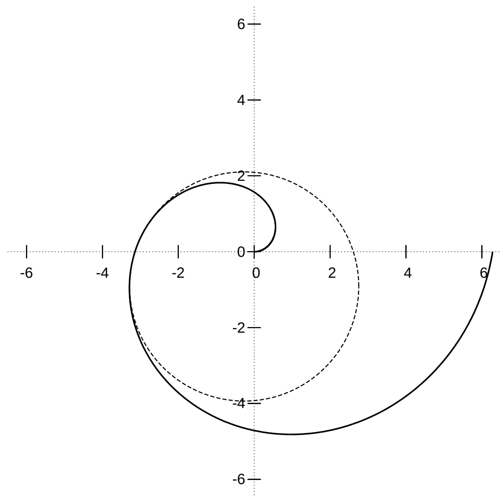
(c) [**10 points**] Let R be the region in the first quadrant of the xy-plane that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$. Find the volume of the solid that lies above R and below the plane $z = y$.



6. [**10 points**] Find the radius of curvature of the Archimedean spiral

$$x = t\cos t, \quad y = t\sin t$$

at the point $(-\pi, 0)$. (You are finding the radius of the circle shown in the figure below.)



7. [**8 points**] Evaluate the integral $\displaystyle\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx.$

8. [**10 points**] A lamina occupies the region in the xy-plane bounded by the lines $x = 1$, $x = 2$, $y = ax$, and $y = 2ax$ for some positive number a. The lamina has density function $\rho(x, y) = \frac{1}{x} + \frac{1}{y^2}$. Find the value of a that minimizes the mass of the lamina.

9. [**10 points**] Find three positive numbers x, y, and z whose sum is 100 and for which the product

$$xy^2z^3$$

is maximum.