Math 126 Challenge Problems/Solutions

Problems Posted 12/3/2013

1. Use Taylor series to prove *Euler's formula*,

$$e^{ix} = \cos x + i \sin x.$$

Use this to determine the five complex 5th roots of 1. (That is, complex numbers z such that $z^5 = 1$.)

2. Use Euler's formula to prove *Euler's identity*,

$$e^{i\pi} + 1 = 0.$$

Also use Euler's formula to show

$$8\cos(20^\circ)^3 - 6\cos(20^\circ) - 1 = 0.$$

(This expression comes up in proving the impossibility of trisecting a 60° angle using only a compass and straightedge. One may show that such a trisection would force the polynomial $8x^3 - 6x - 1$ to have a rational root; it doesn't.)