An Introduction to Pure Mathematics Research

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Slides: http://www.math.ucsd.edu/~jswanson/talks/2021_GRM.pdf

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Outline

- A Bird's-Eye View
- Combinatorics
  - Generating Functions
- Research
  - Algebraic and Analytic Combinatorics
  - A Friendly Example

A Bird's-Eye View

Q What is pure mathematics?

A Any branch of mathematics which is primarily concerned with abstract truth rather than direct applications.

Ex

- Number theory
- Algebraic geometry
- Differential geometry
- Set theory
- $\boxed{\text{Combinatorics}}$
- Probability and Statistics
- Analysis/PDE's
- Optimization
- Mathematical physics

⋮ (many more!)

pick one in grad school!

(See ams.org/msc)

Combinatorics

What is combinatorics?

$\boxed{\text{Def}}$

- "The math of counting things."
- Sara Billey: "Combinatorics is the nanotechnology of math."

$\boxed{\text{Ex}}$ How many ways are there to rearrange the letters in the word COMBINATORICS?

$\boxed{\text{Solution}}$ 778,377,600

Combinatorics

Many further branches:

- Enumerative combinatorics
- Algebraic combinatorics
- Analytic combinatorics
- Extremal combinatorics
- Geometric combinatorics
- Combinatorial optimization
- Computational geometry

⋮ (and more!)

Generating Functions

There is an intimate relationship between algebra and counting. This is frequently encoded in generating functions.

Ex A new Dungeons and Dragons character's Int is obtained by summing 3 6-sided dice. How should we roleplay 9 Int?

```
(x + x^2 + x^3 + x^4 + x^5 + x^6)^3 / 6.0^3 // Expand
```

$0.00462963\,x^3 + 0.0138889\,x^4 + 0.0277778\,x^5 + 0.0462963\,x^6 + 0.0694444\,x^7 + 0.0972222\,x^8 + \underline{0.115741\,x^9} + 0.125\,x^{10} + 0.125\,x^{11} + 0.115741\,x^{12} + 0.0972222\,x^{13} + 0.0694444\,x^{14} + 0.0462963\,x^{15} + 0.0277778\,x^{16} + 0.0138889\,x^{17} + 0.00462963\,x^{18}$

top of the bottom 37.5%

Algebraic and Analytic Combinatorics

Much of my research involves symmetric function theory and tableaux combinatorics underlying the representation theory of Coxeter groups, complex reflection groups, and coinvariant algebras.

One direction involves asymptotics of Mahonian statistics obtained by combining probabilistic techniques and torturing generating functions.

(None of this is reasonable to discuss in detail here!)

A Friendly Example

Problem (Manuel Abellanas) Does every set of unit disks have a conveyor belt?

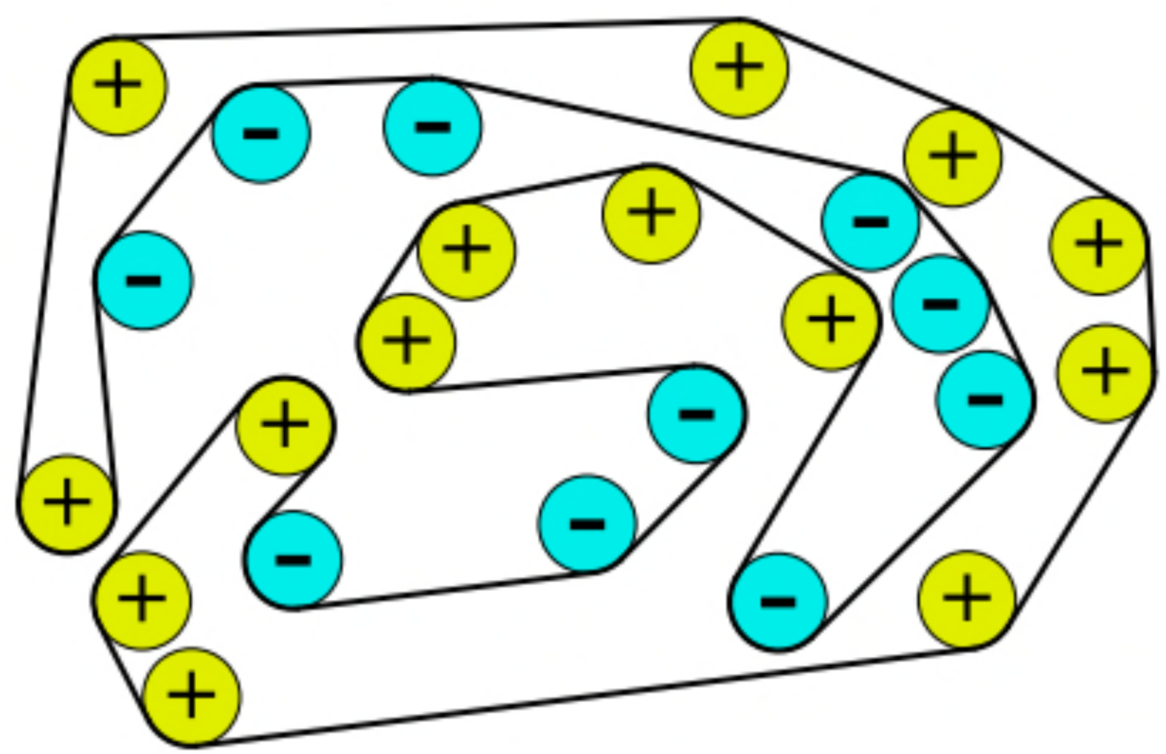


FIGURE 1. A conveyor belt on 24 nonoverlapping unit disks. The colors and markings of the disks indicate their orientations.

A Friendly Example

Thm (BBDDEFGGMS, 2020)

If the disk centers $(x_1, y_1), \ldots, (x_n, y_n)$ are xy-monotone, i.e.

$$x_1 < x_2 < \cdots < x_n$$

and $$y_1 < y_2 < \cdots < y_n,$$

then there is a conveyor belt.

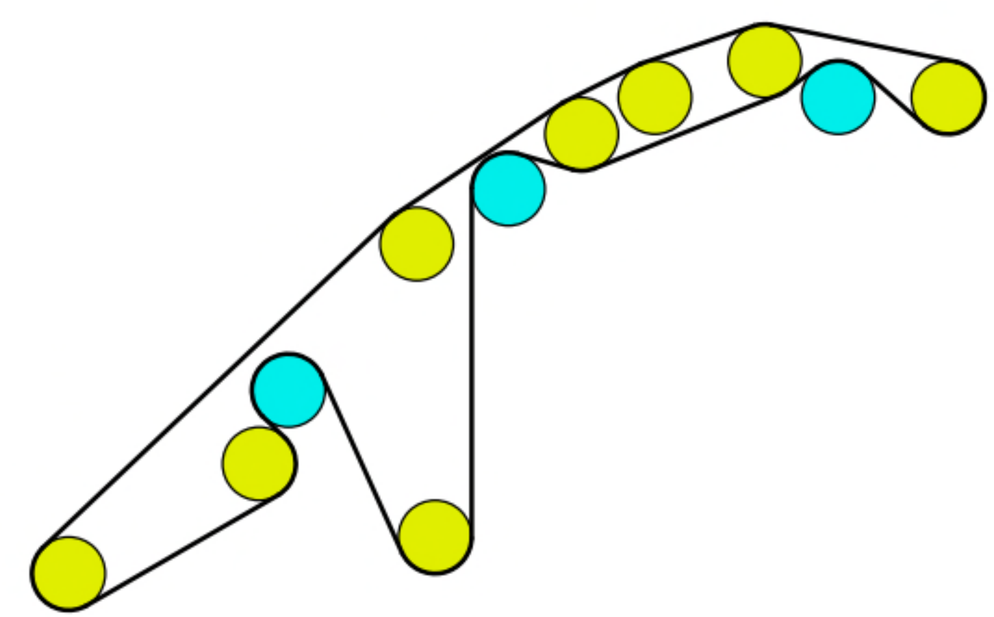
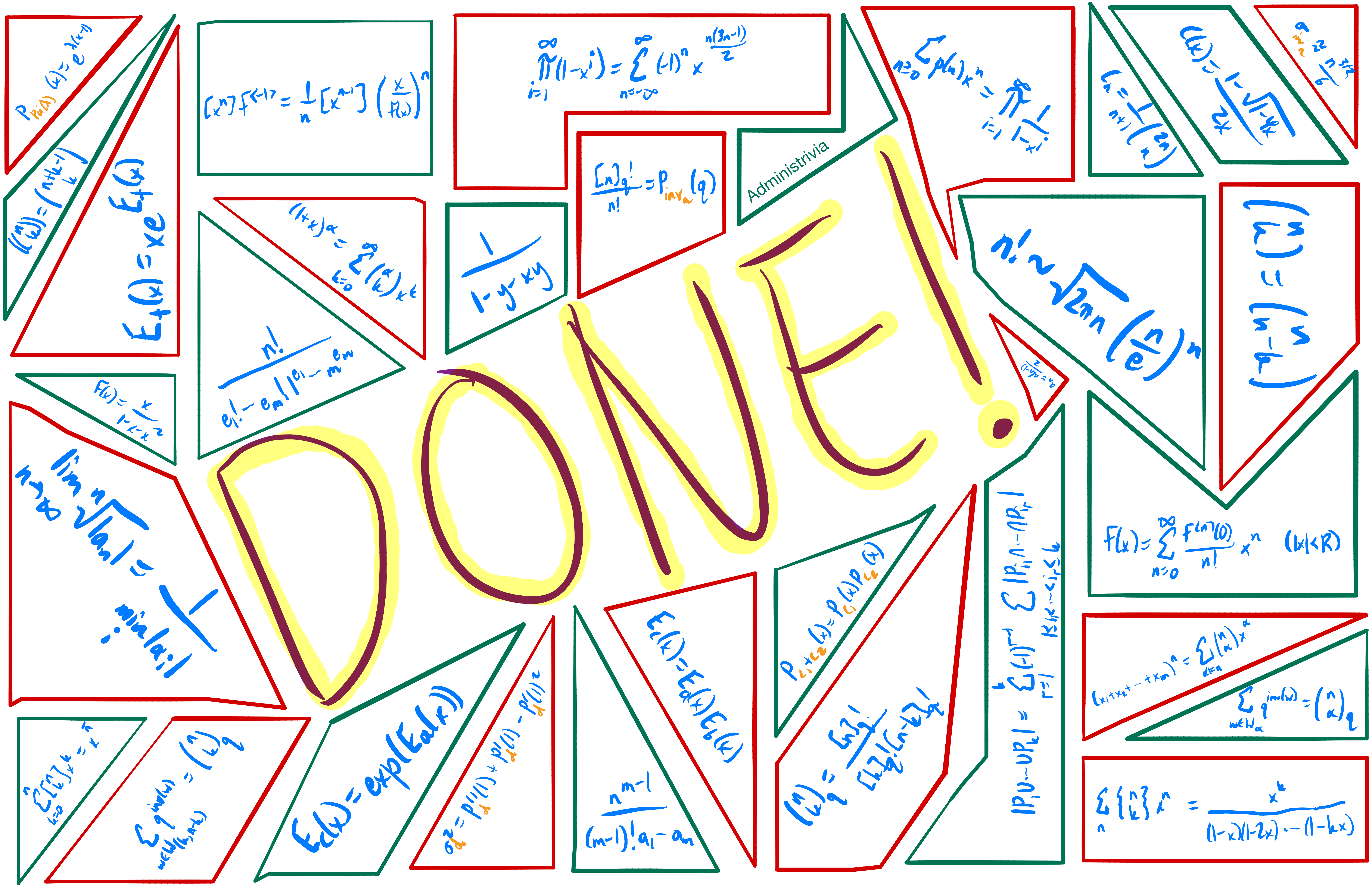


FIGURE 6. Larger example of the conveyor belt produced by the proof of Theorem 5.

References

- [BBDDEFGGMS20] M. Baird, S.C. Billey, E.D. Demaine, M.L. Demaine, D. Eppstein, S. Fekete, G. Gordon, S. Griffin, J.S.B. Mitchell, J.P. Swanson "Existence and hardness of conveyor belts" Electron. J. Combin. 27 (2020), no. 4. Paper 4.25. 21 pages.

- [S20] B. Sagan "Combinatorics: The Art of Counting" Graduate Studies in Mathematics 210 (2020)



DONE!
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